Trigonometric Formulas

$$\cos^2 x + \sin^2 x = 1 \qquad \cos^2 x = \frac{1}{1+\tan^2 x} \qquad \sin^2 x = \frac{\tan^2 x}{1+\tan^2 x}$$

$$\cos(a - b) = \cos a.\cos b + \sin a.\sin b$$

$$\cos(a + b) = \cos a.\cos b - \sin a.\sin b \implies \begin{cases} \cos 2a = \cos^2 a - \sin^2 a \\ \\ \begin{rcases} \cos 2a = 2\cos^2 a - 1 \\ \cos 2a = 1 - 2\sin^2 a \end{rcases} \implies \begin{cases} \cos^2 a = \dfrac{1+\cos 2a}{2} \\ \sin^2 a = \dfrac{1-\cos 2a}{2} \end{cases} \\ \\ \cos 2a = \dfrac{1-\tan^2 a}{1+\tan^2 a} \end{cases}$$

$$\sin(a - b) = \sin a.\cos b - \sin b.\cos a$$

$$\sin(a + b) = \sin a.\cos b + \sin b.\cos a \implies \begin{cases} \sin 2a = 2\sin a.\cos a \\ \sin 2a = \dfrac{2\tan a}{1+\tan^2 a} \end{cases}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1+\tan a.\tan b}$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1-\tan a.\tan b} \implies \tan 2a = \frac{2\tan a}{1-\tan^2 a}$$

$$\cos a.\cos b = \frac{1}{2}[\cos(a - b) + \cos(a + b)] \implies \cos p + \cos q = 2\cos\frac{p+q}{2}.\cos\frac{p-q}{2}$$

$$\sin a.\sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)] \implies \cos p - \cos q = -2\sin\frac{p+q}{2}.\sin\frac{p-q}{2}$$

$$\sin a.\cos b = \frac{1}{2}[\sin(a - b) + \sin(a + b)] \implies \begin{cases} \sin p + \sin q = 2\sin\dfrac{p+q}{2}.\cos\dfrac{p-q}{2} \\ \sin p - \sin q = 2\cos\dfrac{p+q}{2}.\sin\dfrac{p-q}{2} \end{cases}$$

$$\sin 3a = 3\sin a - 4\sin^3 a \quad \text{and} \quad \cos 3a = 4\cos^3 a - 3\cos a$$

$$\begin{cases} \cos(-\alpha) = \cos\alpha \\ \sin(-\alpha) = -\sin\alpha \\ \tan(-\alpha) = -\tan\alpha \end{cases}, \quad \begin{cases} \cos(\pi - \alpha) = -\cos\alpha \\ \sin(\pi - \alpha) = \sin\alpha \\ \tan(\pi - \alpha) = -\tan\alpha \end{cases}, \quad \begin{cases} \cos(\pi + \alpha) = -\cos\alpha \\ \sin(\pi + \alpha) = -\sin\alpha \\ \tan(\pi + \alpha) = \tan\alpha \end{cases}, \quad \begin{cases} \cos(\dfrac{\pi}{2} - \alpha) = \sin\alpha \\ \sin(\dfrac{\pi}{2} - \alpha) = \cos\alpha \\ \tan(\dfrac{\pi}{2} - \alpha) = \cot\alpha \end{cases}$$